Family Name:_____**Given Name:**_____ **I.D.#**_____

MAT3320 Assignment 4

Total: 10 marks. Due date: Tuesday, July 18, on or before 4:00pm.

In MATH Department (585 King Edward), there is a Drop-Box. You need to put your assignment into the box **on or before 4:00pm** on the due date. Late assignments will not be accepted.

1. (2 marks) The solution of Laplace's equation $u_{xx} + u_{yy} = 0$, $0 < x < L, 0 < y < M$ with the boundary conditions $u(x, M) = f(x), u(x, 0) = u(0, y) = u(L, y) = 0$ is given by

$$u(x, y) = \sum_{n=1}^{\infty} b_n \sinh\left(\frac{n\pi y}{L}\right) \sin\left(\frac{n\pi x}{L}\right).$$

Find the solution of Laplace's equation $u_{xx} + u_{yy} = 0$ within

$$R = \{(x, y) : 0 < x < 3, 0 < y < 2\}$$

with

$$BC: \quad u(x, 0) = 0, u(x, 2) = x - 3, u(0, y) = 0, u(3, y) = 0.$$

2. (3 marks) The solution of $u_{tt} = c^2(u_{xx} + u_{yy}), \quad (x, y) \in R = [0, a] \times [0, b], t > 0$, subject to $BC : u(x, y, t) = 0$ for $t > 0$ and $(x, y) \in \partial R$ (boundary of R), is

$$u(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (A_{mn} \cos[\lambda_{mn} t] + B_{mn} \sin[\lambda_{mn} t]) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b},$$

where $\lambda_{mn} = c\pi \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$.

Solve $u_{tt} = 25(u_{xx} + u_{yy}), \quad (x, y) \in R = [0, 3] \times [0, 2], t > 0$, subject to

$BC : u(x, y, t) = 0$ for $t > 0$ and $(x, y) \in \partial R$,

$ICs : u(x, y, 0) = 0, u_t(x, y, 0) = \pi \sin(3\pi x) \sin(4\pi y), (x, y) \in R.$

3. (2 marks) Find the solution $u(r, \varphi)$ of

$$u_{rr} + \frac{2}{r} u_r + \frac{1}{r^2} u_{\varphi\varphi} + \frac{\cot \varphi}{r^2} u_\varphi = 0, \quad r < 4,$$

such that $u(0, \varphi)$ is bounded, and $u(4, \varphi) = \cos(2\varphi)$.

4. (3 marks) Evaluate

$$\int_0^3 x^5 J_2(2x)dx.$$

Remark. Your solution should be a linear combination of $J_3(6)$ and $J_4(6)$.